Zoo of Quantum Halos

Yusuke Nishida (Tokyo Tech)

Hadrons and Nuclear Physics
meet ultracold atoms:
a French Japanese workshop @ IHP

Jan. 29 - Feb. 2 (2018)

Plan of this talk

1. Introduction to quantum halos
   - Efimov effect
   - Super Efimov effect
2. Semi-super Efimov effect
   - ~~Model analysis~~
   - RG analysis and universality
3. Classification of quantum halos
   - Even more universality classes ?
4. Summary

Ref: Y. Nishida, "Semi-super Efimov effect of two-dimensional bosons at a three-body resonance," Phys. Rev. Lett. 118, 230601 (2017)

Y. Sekino & Y. Nishida, "Quantum droplet of one-dimensional bosons with a three-body attraction," Phys. Rev. A 97, 011602(R) (2018)

Introduction

Quantum halos

REVIEWS OF MODERN PHYSICS, VOLUME 76, JANUARY 2004

Structure and reactions of quantum halos

A. S. Jensen, K. Riisager, and D. V. Fedorov

Department of Physics and Astronomy, Aarhus University, DK-8000 Aarhus C, Denmark

E. Garrido

Instituto de Estructura de la Materia, CSIC, Serrano 123, E-28006 Madrid, Spain

(Published 5 February 2004)

This article provides an overview of the basic principles of the physics of quantum halo systems, defined as bound states of clusters of particles with a radius extending well into classically forbidden regions. Exploiting the consequences of this definition, the authors derive the conditions for occurrence in terms of the number of clusters, binding energy, angular momentum, cluster charges, and excitation energy. All these quantities must be small. The article discusses the transitions between different cluster divisions and the importance of thresholds for cluster or particle decay, with particular attention to the Efimov effect and the related exotic states. The pertinent properties can be described by the use of dim[illegible] properties can be distinguished, as shown in a series of examples selected from nuclear, atomic, and molecular systems. The neutron dripline is especial[illegible] for atoms. For molecules, in which the cluster division comes naturally, a wider range of possibilities exists. Halos in two dimensions have very different properties, and their states are easily spatially extended, whereas Borromean systems are unlikely and spatially confined. The Efimov effect and the Thomas collapse occur only for dimensions between 2.3 and 3.8 and thus not for 2. High-energy reactions directly probe

size ≫ potential range ~ r_0

⇒ no classical counterparts

⇒ universal properties

Quantum halos

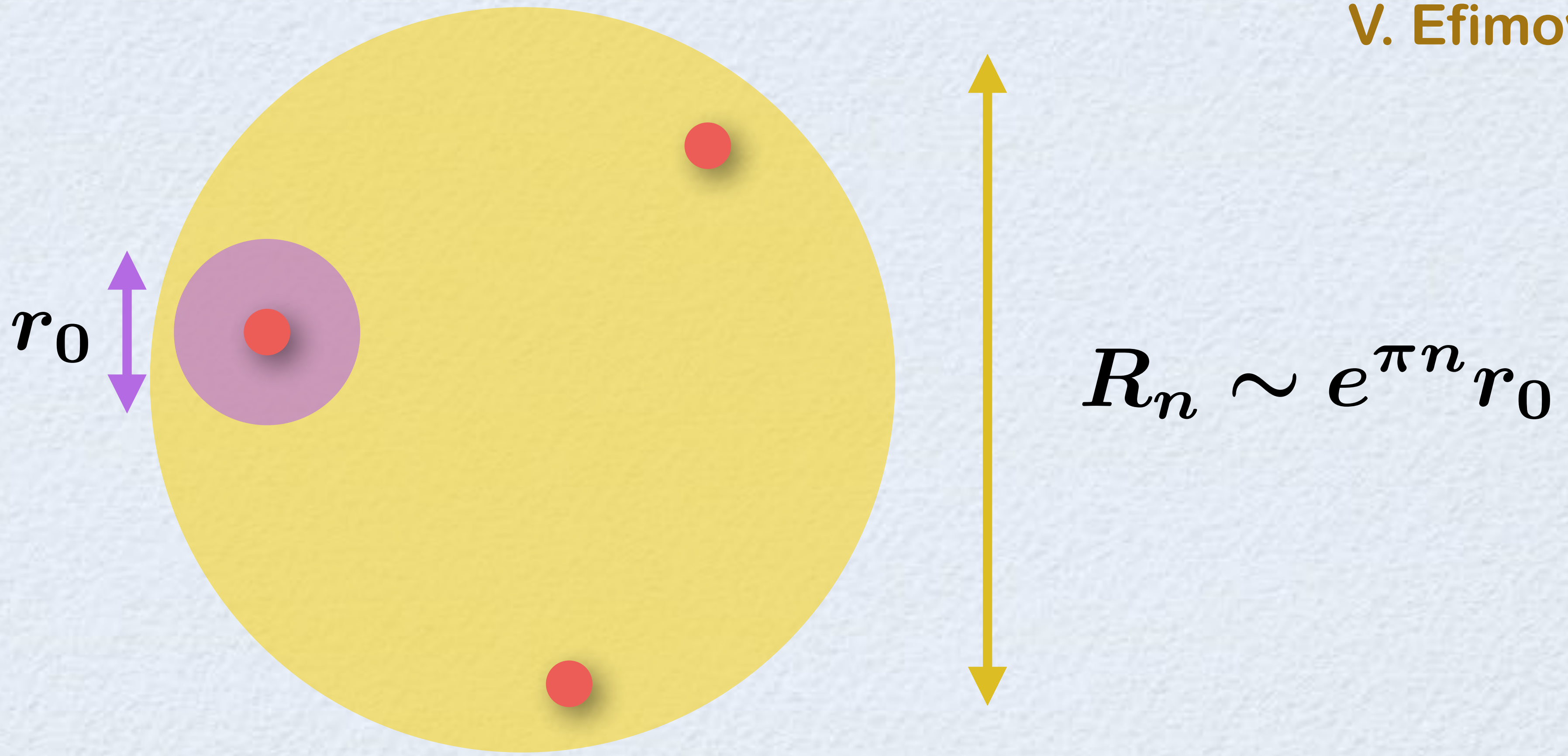
Efimov effect

- ✓ 3 bosons
- ✓ 3 dimensions
- ✓ s-wave resonance

→ Infinite bound states with universal scaling

$$E_n \sim e^{-2\pi n}$$

V. Efimov, PLB (1970)



Quantum halos can be arbitrarily large for n>>1 !

Quantum halos

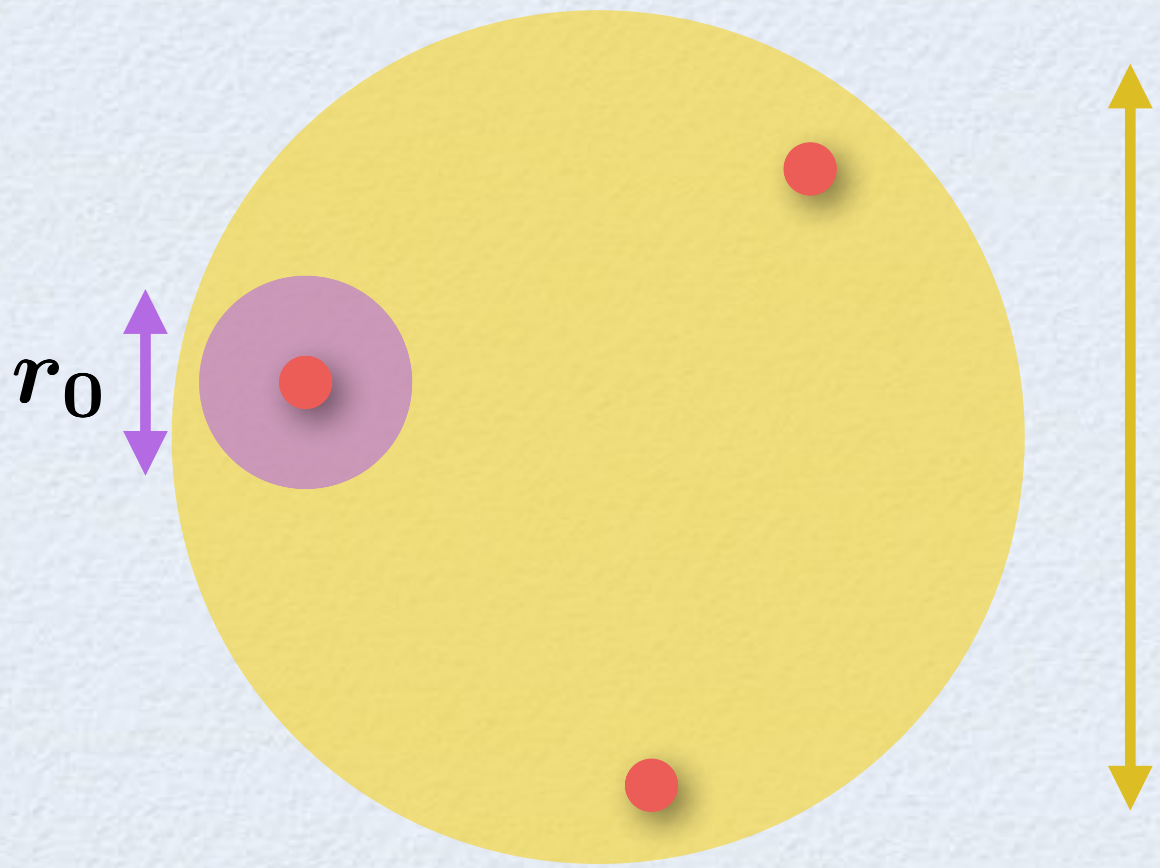
Super Efimov effect

- ✓ 3 fermions
- ✓ 2 dimensions
- ✓ p-wave resonance

Infinite bound states with universal scaling

$$E_n \sim e^{-2e^{3\pi n/4}}$$

Y. Nishida, S. Moroz, D. T. Son, PRL (2013)



$$R_n \sim e^{e^{3\pi n/4}} r_0$$

Quantum halos can be arbitrarily large for n>>1 !

Quantum halos

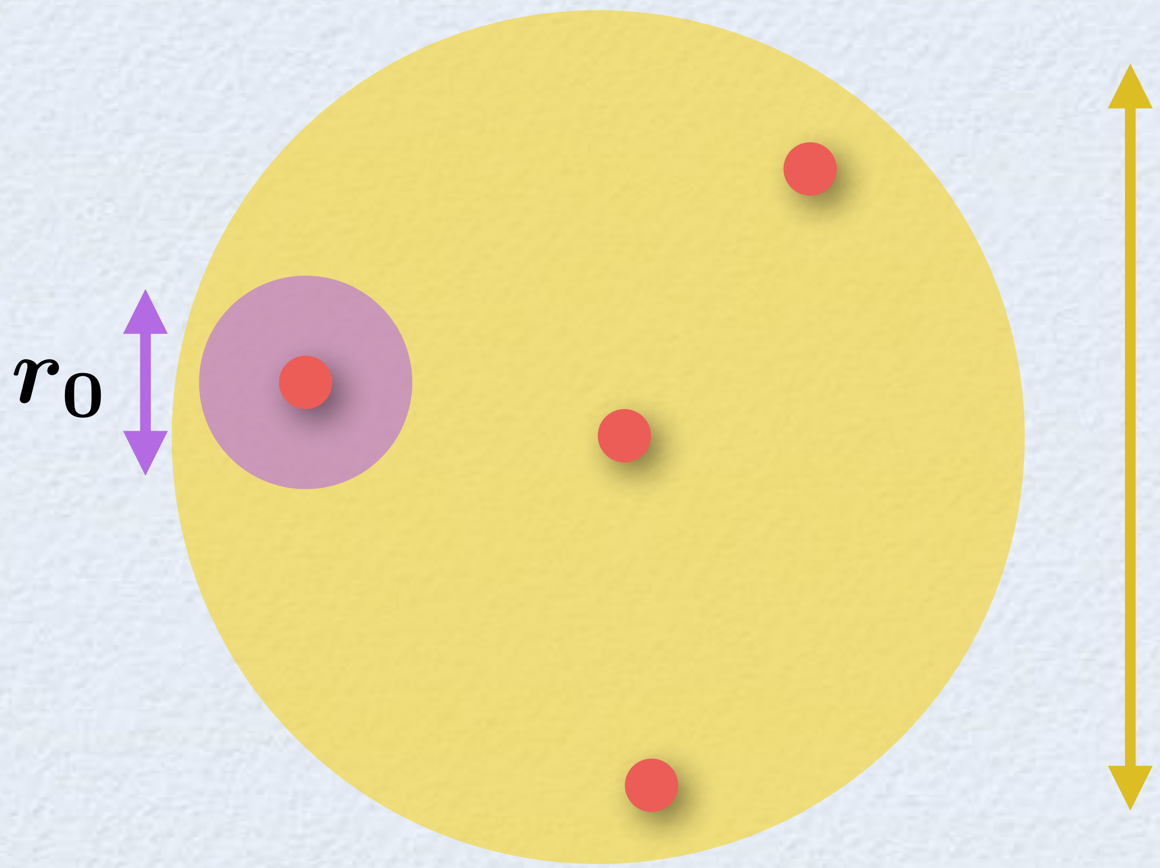
Semi-super Efimov effect

- ✓ 4 bosons
- ✓ 2 dimensions
- ✓ 3-body resonance

➡ Infinite bound states with universal scaling

$$E_n \sim e^{-2(\pi n)^2/27}$$

Y. Nishida, PRL (2017)



$$R_n \sim e^{(\pi n)^2/27} r_0$$

Quantum halos can be arbitrarily large for n>>1 !

Semi-super Efimov effect

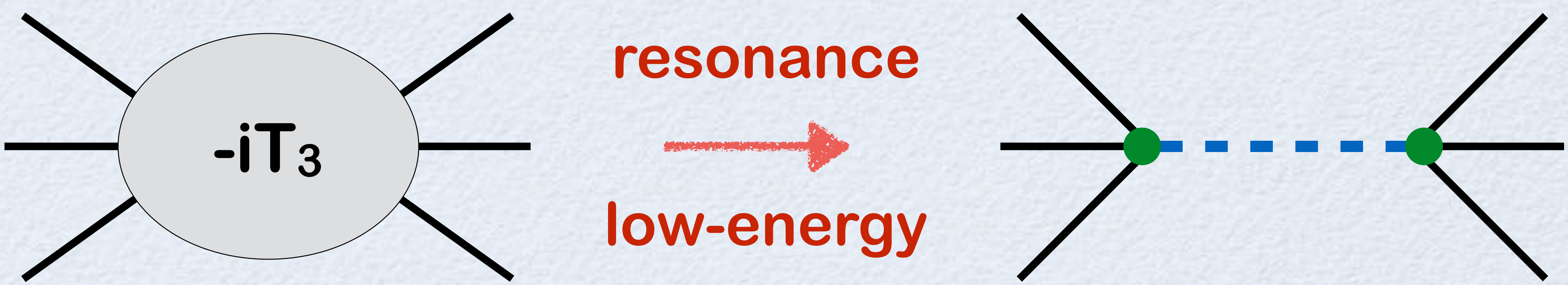
RG analysis

3-body scattering T-matrix in effective range expansion

$$T_3(\varepsilon) = \frac{\pi^2}{\cancel{\frac{1}{A}} + \varepsilon \ln\left(\frac{\Lambda}{\sqrt{-\varepsilon}}\right) + \cancel{O(\varepsilon^2)}} \to \frac{\pi^2}{\ln\left(\frac{\Lambda}{\sqrt{-\varepsilon}}\right)} \times \frac{1}{E - \frac{k^2}{6m} + i0^+}$$

(collision energy $\varepsilon = E - \frac{k^2}{6m} + i0^+$)

⇒ Propagation of a point-like trimer



Singular wave function at origin : $\Psi(r_1, r_2, r_3) \underset{R \to 0}{\to} \frac{C}{R^2}$

Cf. p-wave in 2D, s-wave in 4D

RG analysis

Low-energy effective field theory

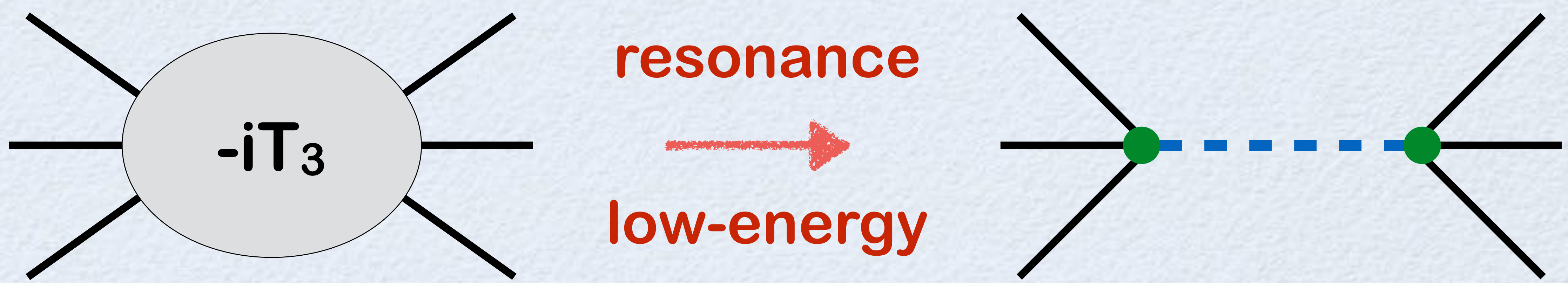
$$\mathcal{L} = \phi^\dagger\left(i\partial_t + \frac{\nabla^2}{2m}\right)\phi + \underline{v_2\phi^\dagger\phi^\dagger\phi\phi}$$

2-body interaction

$$+\,\Phi^\dagger\left(i\partial_t + \frac{\nabla^2}{6m}\right)\Phi + \underline{g\,\Phi^\dagger\phi\phi\phi + g\,\phi^\dagger\phi^\dagger\phi^\dagger\Phi}$$

3-body int.

⇒ Propagation of a point-like trimer



Singular wave function at origin : $\Psi(r_1, r_2, r_3) \underset{R\to 0}{\to} \dfrac{C}{R^2}$

Cf. p-wave in 2D, s-wave in 4D

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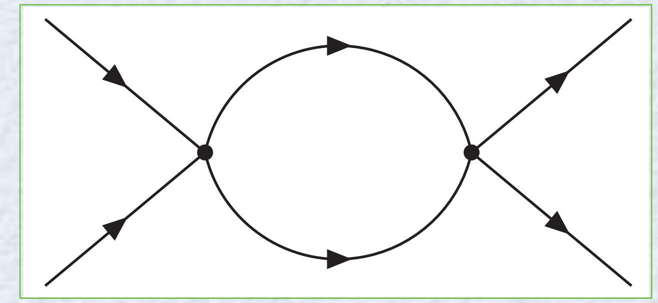
Low-energy effective field theory



$$\mathcal{L} = \phi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \phi + \underline{v_2 \phi^\dagger \phi^\dagger \phi \phi}$$

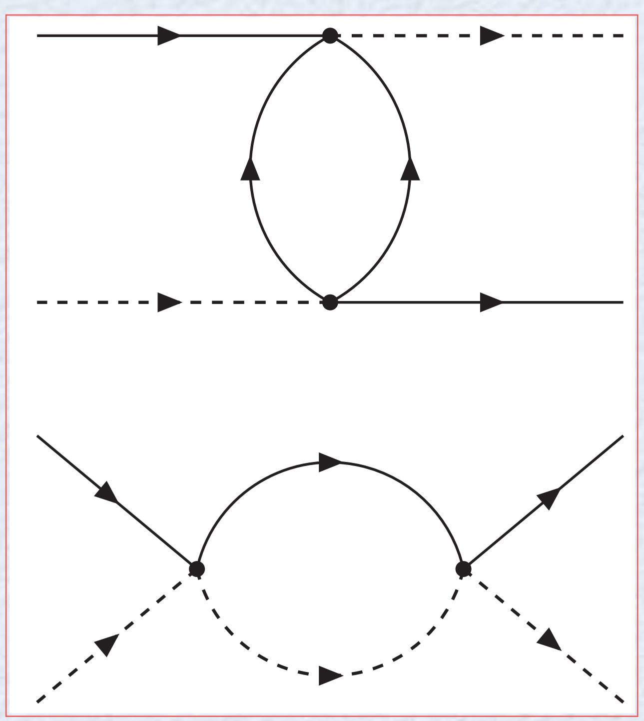
2-body interaction

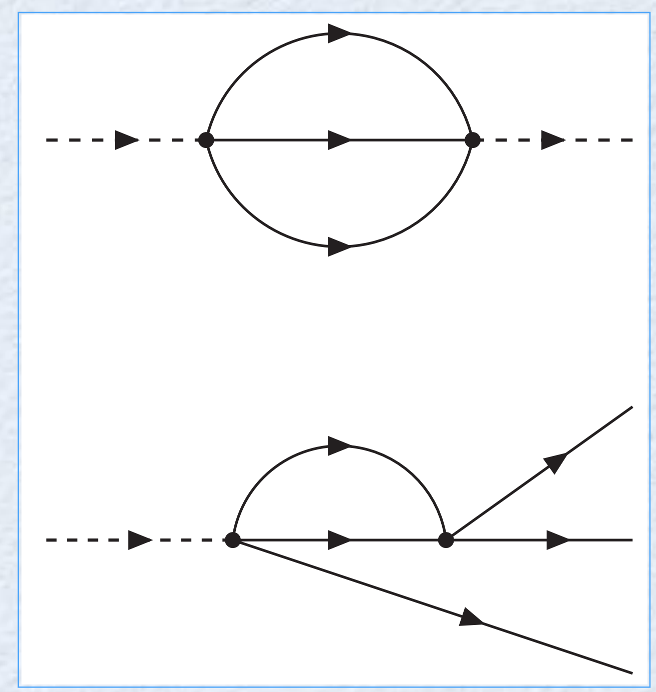
$$+ \Phi^\dagger \left(i\partial_t + \frac{\nabla^2}{6m} \right) \Phi + \underline{g\, \Phi^\dagger \phi\phi\phi + g\, \phi^\dagger \phi^\dagger \phi^\dagger \Phi}$$

3-body int.

$$+ \underline{v_4 \phi^\dagger \Phi^\dagger \Phi \phi} + v_6 \Phi^\dagger \Phi^\dagger \Phi \Phi$$

4-body int.





coupling constants

⇒ running couplings

RG analysis

Low-energy effective field theory

$$\mathcal{L} = \phi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \phi + \underline{v_2 \phi^\dagger \phi^\dagger \phi \phi}$$

2-body interaction

$$+ \Phi^\dagger \left(i\partial_t + \frac{\nabla^2}{6m} \right) \Phi + \underline{g\, \Phi^\dagger \phi\phi\phi + g\, \phi^\dagger \phi^\dagger \phi^\dagger \Phi}$$

3-body int.

$$+ \underline{v_4 \phi^\dagger \Phi^\dagger \Phi \phi} + v_6 \Phi^\dagger \Phi^\dagger \Phi \Phi$$

4-body int.

⇒ RG equations

$(s \equiv \ln \Lambda/\kappa)$

- $$\frac{dv_2}{ds} = \frac{v_2^2}{\pi}$$
- $$\frac{dg}{ds} = -\frac{g^3}{2\pi^2} + \frac{3gv_2}{\pi}$$
- $$\frac{dv_4}{ds} = \frac{9g^2}{\pi} + \frac{3v_4^2}{4\pi} - \frac{g^2 v_4}{\pi^2} + O(g^2 v_2)$$

RG analysis

Solution for v_2=0 in the low-energy limit $s \equiv \ln \Lambda/\kappa \to \infty$

2-body ▶ $$\frac{dv_2}{ds} = \frac{v_2^2}{\pi} \quad \Rightarrow \quad v_2(s) = 0$$

3-body ▶ $$\frac{dg}{ds} = -\frac{g^3}{2\pi^2} + \cancel{\frac{3gv_2}{\pi}} \quad \Rightarrow \quad g^2(s) \to \frac{\pi^2}{s}$$

4-body ▶ $$\frac{dv_4}{ds} = \frac{9g^2}{\pi} + \frac{3v_4^2}{4\pi} - \frac{g^2 v_4}{\pi^2} + \cancel{O(g^2 v_2)}$$

$$\Rightarrow \quad v_4(s) \to -\frac{6\pi}{\sqrt{3s}} \cot\left(\sqrt{27s} - \theta\right)$$

diverges at $\sqrt{27s} = \pi n + \theta$

⇒ 4-body binding energies at $\kappa_n \to e^{-(\pi n + \theta)^2/27}\Lambda$

Semi-super Efimov effect !

RG analysis

Solution for $v_2 \neq 0$ in the low-energy limit $s \equiv \ln \Lambda/\kappa \to \infty$

2-body ▶ $$\frac{dv_2}{ds} = \frac{v_2^2}{\pi} \quad \Rightarrow \quad v_2(s) \to -\frac{\pi}{s}$$

3-body ▶ $$\frac{dg}{ds} = -\frac{g^3}{2\pi^2} + \frac{3gv_2}{\pi} \quad \Rightarrow \quad g^2(s) \to \frac{C}{s^6}$$

4-body ▶ $$\frac{dv_4}{ds} = \cancel{\frac{9g^2}{\pi}} + \frac{3v_4^2}{4\pi} - \cancel{\frac{g^2 v_4}{\pi^2}} + \cancel{O(g^2 v_2)}$$

$$\Rightarrow \quad v_4(s) \to -\frac{4\pi}{3s}$$

Semi-super Efimov effect disappears

⇒ Double fine-tunings of 2-body and 3-body int. are needed but possible with ultracold atoms !

D.S.Petrov, PRL (2014); A.J.Daley, J. Simon, PRA (2014);
D.S.Petrov, PRA (2014); S.Paul, P.R.Johnson, E.Tiesinga, PRA (2016)

Quantum Halo Zoo

Universal scaling laws

Spinless bosons

- 3D + 2-body res. ⇒ $\kappa_n \sim e^{-\pi n/1.006}$ V.Efimov, PLB (1970)
- 2D + 3-body res. ⇒ $\kappa_n \sim e^{-(\pi n)^2/27}$ Y.Nishida, PRL (2017)
- 1D + 4-body res. ⇒ $\kappa_n \sim e^{-\pi n/1.247}$ Y.Nishida, D.T.Son, PRA (2010)

Spinless fermions

- 2D + 2-body res. ⇒ $\kappa_n \sim e^{-e^{3\pi n/4}}$ Y.Nishida, S.Moroz, D.T.Son, PRL (2013)
- unknown in 3D & 1D

Interesting hierarchy & interplay among statistics, dimensionality, required interaction, and emergent universal scaling laws

Universal scaling laws

Spinless bosons

- 3D + 2-body res. ⇒ Efimov effect
- 2D + 3-body res. ⇒ Semi-super Efimov effect
- 1D + 4-body res. ⇒ Efimov effect

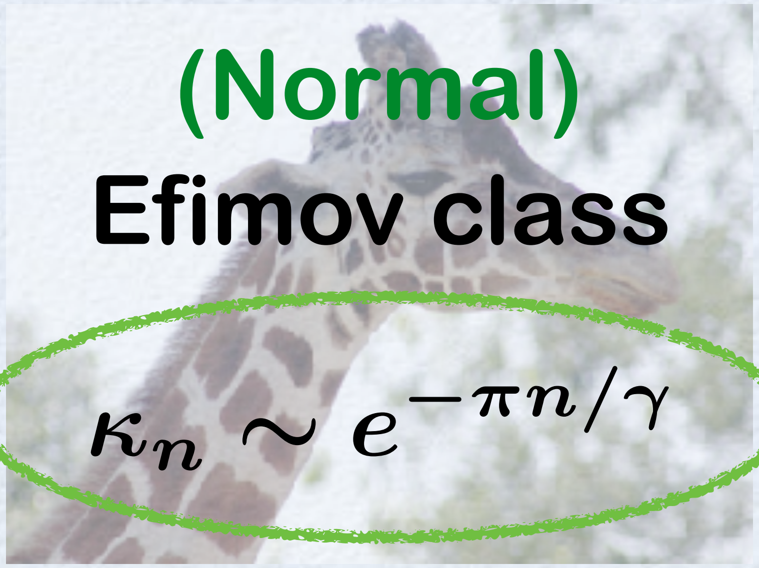
Spinless fermions

- 2D + 2-body res. ⇒ Super Efimov effect
- unknown in 3D & 1D

Interesting hierarchy & interplay among statistics, dimensionality, required interaction, and emergent universal scaling laws

Classification

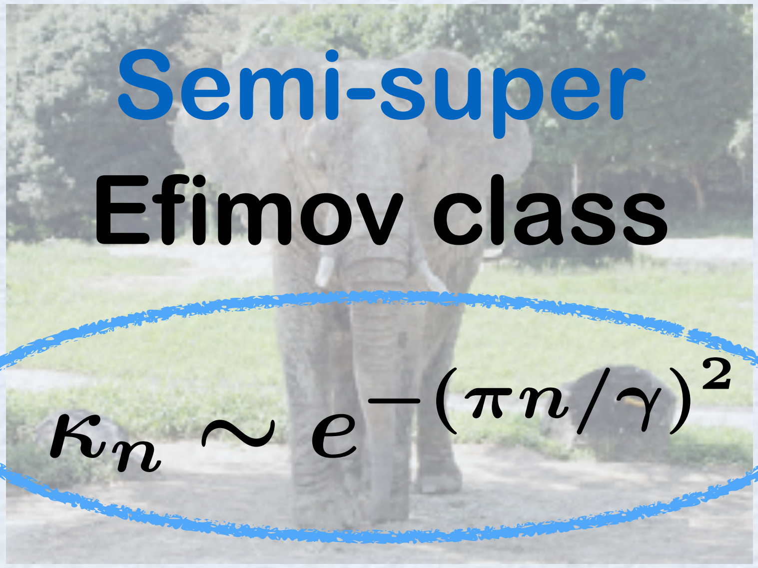
Known universal scaling laws are classified into



(Normal) Efimov class

$$\kappa_n \sim e^{-\pi n/\gamma}$$

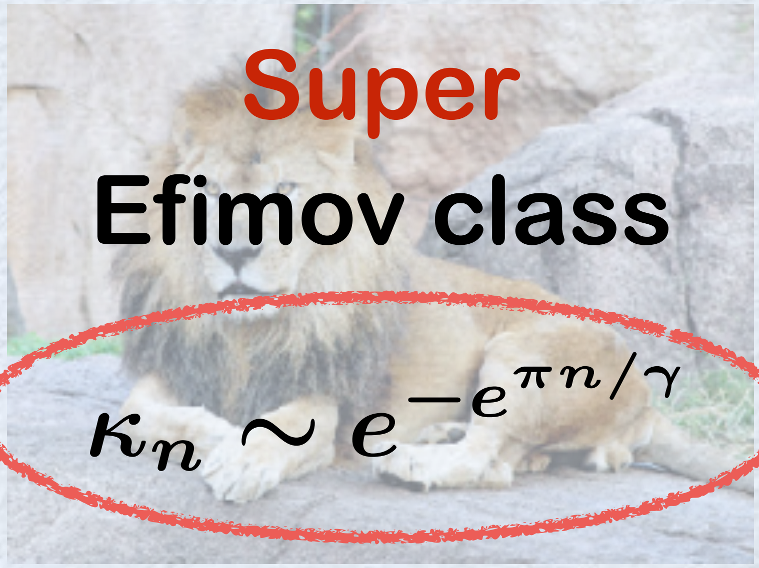
- ✓ 3 bosons in 3D (Efimov, 1970)
- 4 anyons in 2D (Nishida, 2008)
- ✓ 5 bosons in 1D (Nishida, Son, 2010)
- mass-imbalanced 3, 4, 5 fermions in 3D ⟵ next talk ! (Efimov, 1973; Castin et al., 2010 Bazak, Petrov, 2017)
- mixed dimensions (Nishida, Tan, 2008; 2011)



Semi-super Efimov class

$$\kappa_n \sim e^{-(\pi n/\gamma)^2}$$

- ✓ 4 bosons in 2D (Nishida, 2017)
- mixed dimensions (Zhang, Yu, 2017)



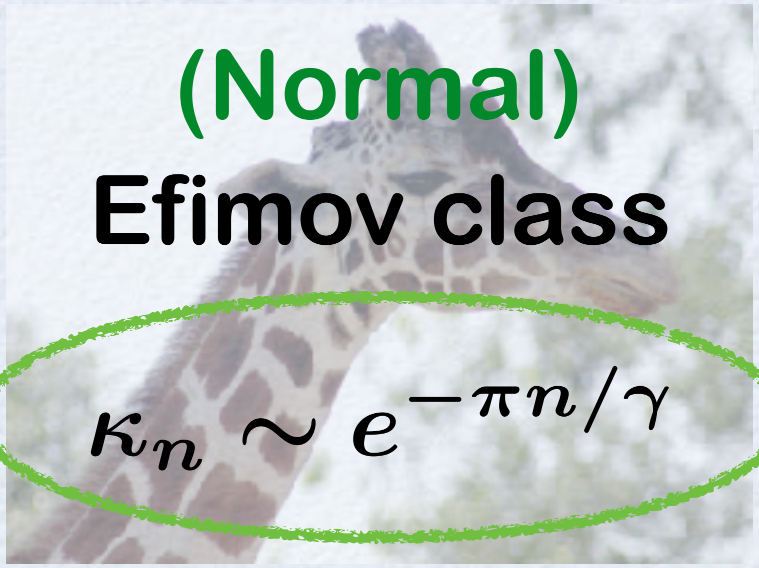
Super Efimov class

$$\kappa_n \sim e^{-e^{\pi n/\gamma}}$$

- ✓ 3 fermions in 2D (Nishida et al., 2013)
- mass-imbalanced bosons / fermions in 2D (Moroz, Nishida, 2014)
- mixed dimensions (Zhang, Yu, 2017)

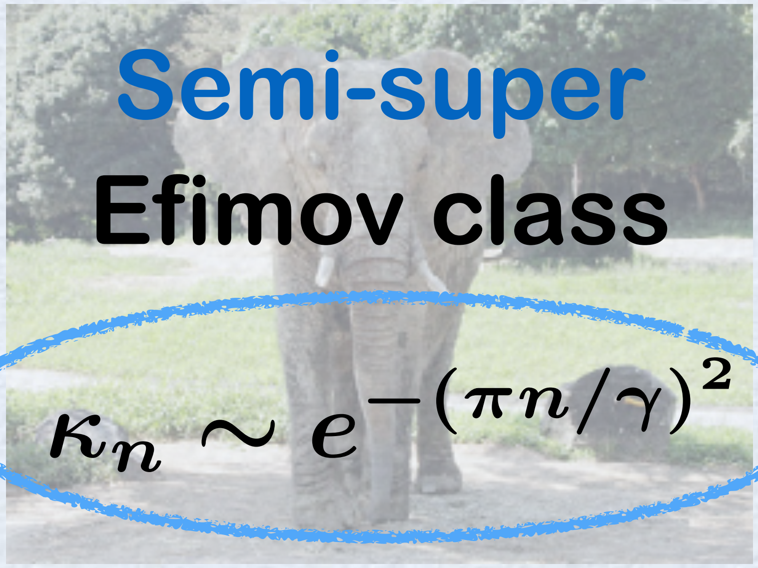
Classification

Known universal scaling laws are classified into



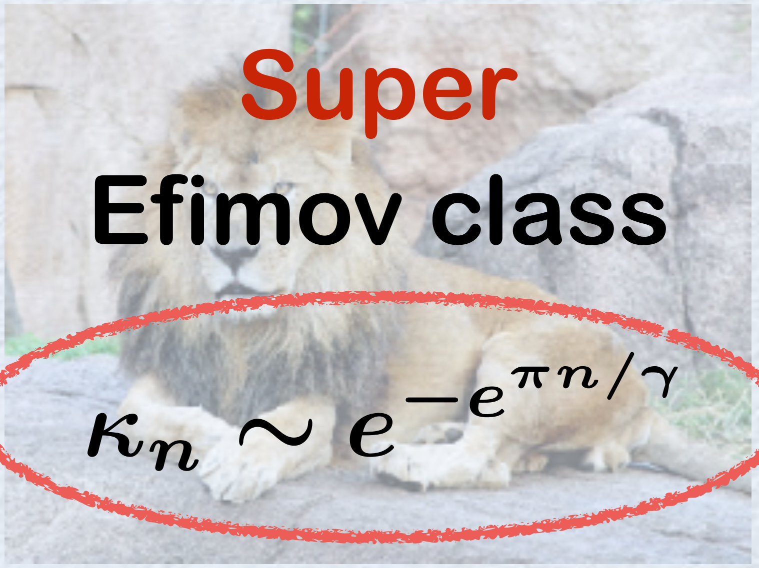
(Normal) Efimov class

$\kappa_n \sim e^{-\pi n/\gamma}$



Semi-super Efimov class

$\kappa_n \sim e^{-(\pi n/\gamma)^2}$



Super Efimov class

$\kappa_n \sim e^{-e^{\pi n/\gamma}}$

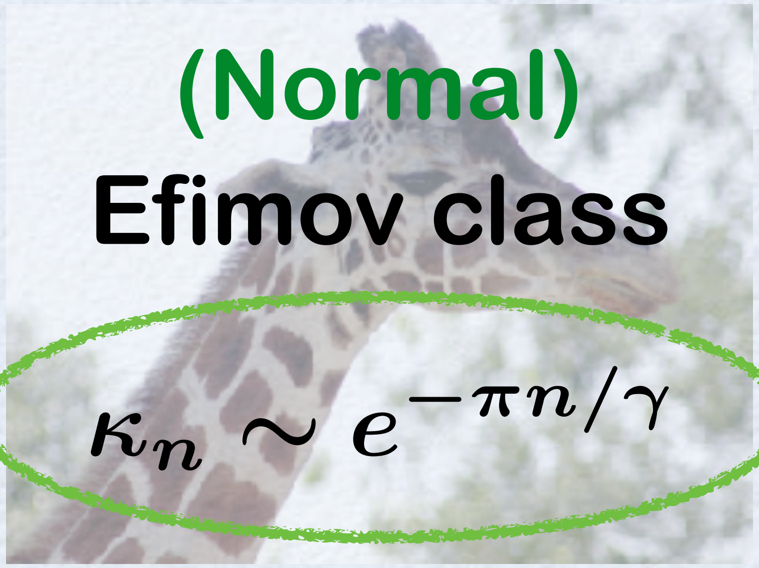
TRiO of few-body universality classes

Q3. Even more universality classes ?

⇒ My speculation is …

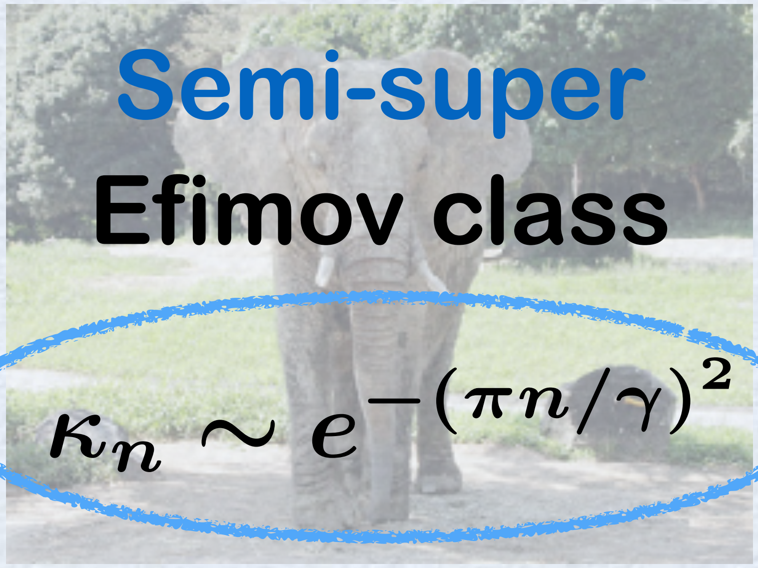
Classification

Known universal scaling laws are classified into



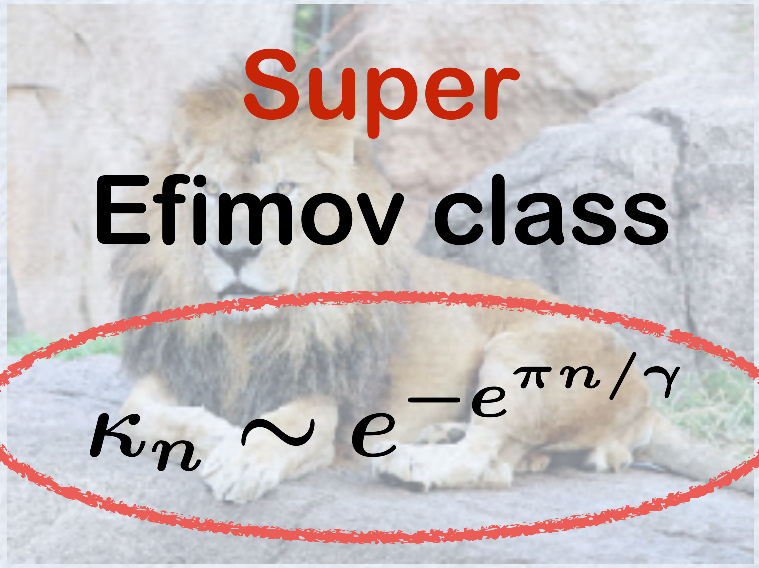
(Normal)
Efimov class

$$\kappa_n \sim e^{-\pi n/\gamma}$$



Semi-super
Efimov class

$$\kappa_n \sim e^{-(\pi n/\gamma)^2}$$



Super
Efimov class

$$\kappa_n \sim e^{-e^{\pi n/\gamma}}$$

$$V(R) \sim \frac{\#}{R^2} \qquad V(R) \sim \frac{\#}{R^2(\ln R)} \qquad V(R) \sim \frac{\#}{R^2(\ln R)^2}$$

hyperspherical potential

no (arbitrarily large) quantum halos if power > 2

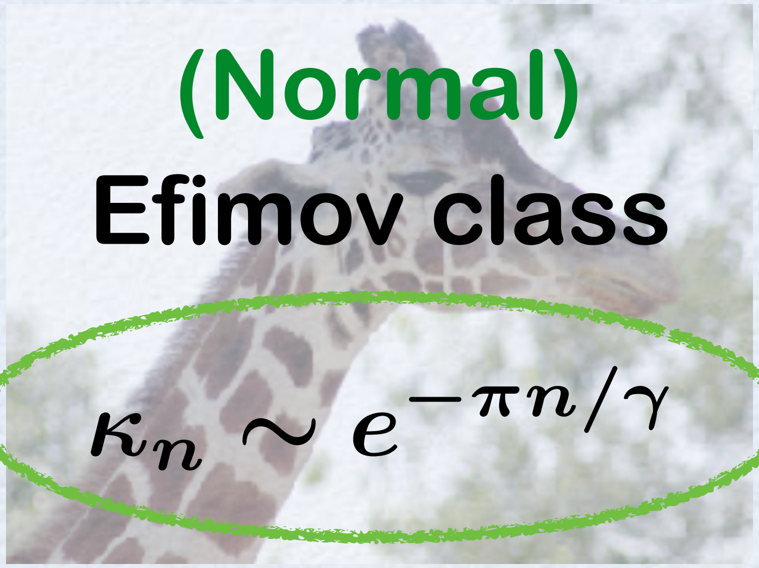
V.Efimov, PLB (1970)

M.A.Efemov, W.P.Schleich, arXiv:1407; arXiv:1511

Volosniev et al., JPB (2014); C.Gao et al., PRA (2015)

Classification

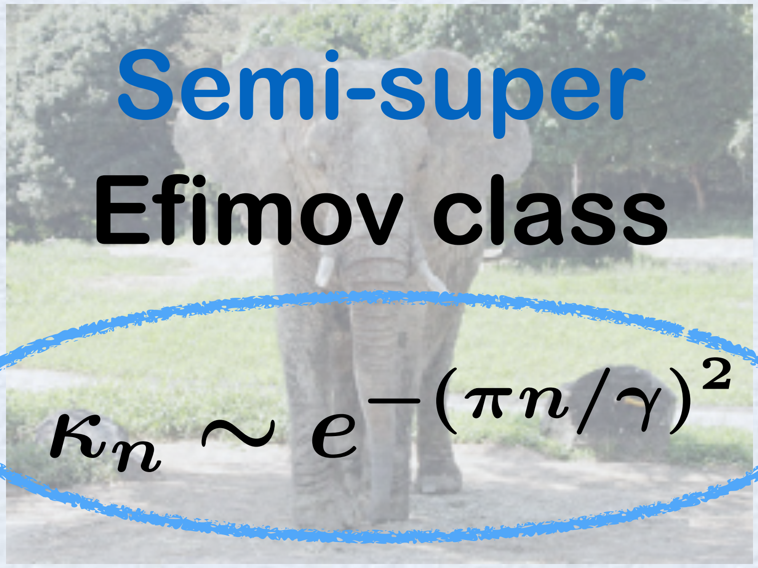
Known universal scaling laws are classified into



(Normal)
Efimov class

$$\kappa_n \sim e^{-\pi n/\gamma}$$

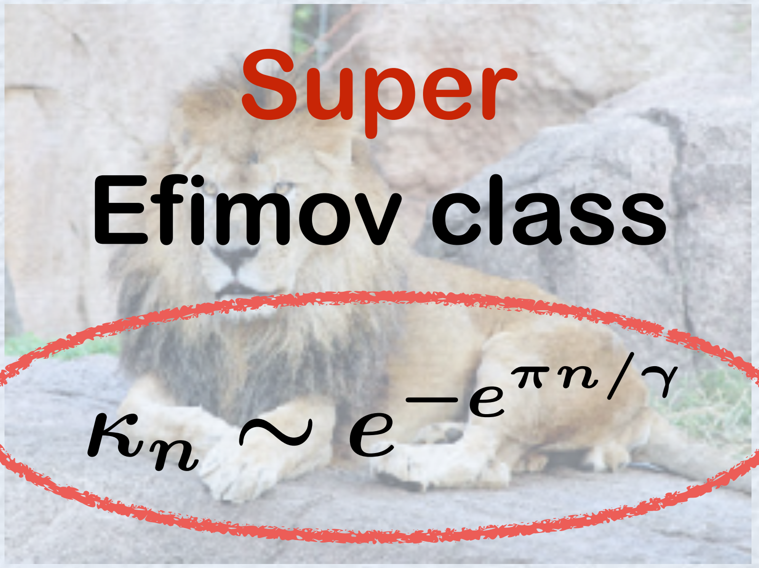
$$V(R) \sim \frac{\#}{R^2}$$



Semi-super
Efimov class

$$\kappa_n \sim e^{-(\pi n/\gamma)^2}$$

$$V(R) \sim \frac{\#}{R^2(\ln R)}$$



Super
Efimov class

$$\kappa_n \sim e^{-e^{\pi n/\gamma}}$$

$$V(R) \sim \frac{\#}{R^2(\ln R)^2}$$

If $V(R) \sim \dfrac{\#}{R^2(\ln R)^2(\ln\ln R)}$

$$\Rightarrow \kappa_n \sim e^{-e^{(\pi n/\gamma)^2}}$$

Semi-hyper
Efimov class

$$V(R) \sim \frac{\#}{R^2(\ln R)^2(\ln\ln R)^2}$$

$$\Rightarrow \kappa_n \sim e^{-e^{e^{\pi n/\gamma}}}$$

Hyper
Efimov class

Classification

Q4. Do semi-hyper and hyper Efimov effects emerge in quantum few-body systems with short-range interactions ?

⇒ I don't know (at this moment).

If $V(R) \sim \dfrac{\#}{R^2(\ln R)^2(\ln\ln R)}$

$$\Rightarrow \kappa_n \sim e^{-e^{(\pi n/\gamma)^2}}$$

Semi-hyper
Efimov class

$V(R) \sim \dfrac{\#}{R^2(\ln R)^2(\ln\ln R)^2}$

$$\Rightarrow \kappa_n \sim e^{-e^{e^{\pi n/\gamma}}}$$

Hyper
Efimov class

“Phase diagram”

Fates of N bosons with few-body attraction

Fully universal (ground and excited states)

	2-body	3-body	4-body
1D	N^3	$e^{8N^2/\sqrt{3}\pi}$	Efimov
2D	$e^{2.148N}$	semi-super Efimov	—
3D	Efimov	—	—

next talk !

Non-universal (?)

Semi-universal

- low-lying states are non-universal
- higher excited states show universal scaling law

McGuire, JMP (1964); Sekino, Nishida, PRA (2018); Nishida, Son, PRA (2010);
Hammer, Son, PRL (2004); Nishida, PRL (2917); Efimov, PLB (1970)

Summary

Known (arbitrarily large) quantum halos are classified into TRiO of few-body universality classes

- ✓ (Normal) Efimov class $\kappa_n \sim e^{-\pi n/\gamma}$
- ✓ Semi-super Efimov class $\kappa_n \sim e^{-(\pi n/\gamma)^2}$
- ✓ Super Efimov class $\kappa_n \sim e^{-e^{\pi n/\gamma}}$

Q. Even more universality classes such as

- ✓ Semi-hyper Efimov class $\kappa_n \sim e^{-e^{(\pi n/\gamma)^2}}$
- ✓ Hyper Efimov class $\kappa_n \sim e^{-e^{e^{\pi n/\gamma}}}$
- ✓ …

emerge in quantum few-body systems with short-range interactions ???